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Further Results of High Reynolds Number Skin-Friction Tests

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Nomenclature

k = grain diameter
M = Mach number
R_θ = Reynolds number based on momentum thickness, $\rho_1 U_1 \theta/\mu_1$
R_x = Reynolds number based on effective flat-plate length, $\rho_1 U_1 X/\mu_1$
U = velocity
U_τ = friction velocity $(\tau_w/\rho_w)^{1/2}$
y = coordinate distance normal to test surface
γ = specific-heat ratio
θ = bouudary-layer momentum thickness
μ = viscosity
ρ = mass density
τ = shearing stress

Subscripts

1 = local conditions at outer edge of boundary layer
w = wall condition

A RECENT experimental boundary-layer study[1] provided a significant extension of the available Reynolds number range of skin-friction data. Subsequent analytical and experimental efforts have provided additional results pertinent to these very high Reynolds number tests. These results concern the evaluation of the Preston-tube method of determining local skin friction for Reynolds numbers up to $R_x = 1.41 \times 10^9$ at $M = 2.8$ and the study of drag effects of uniform grain-type surface roughness.

The relatively simple technique of experimentally determining local skin friction in pipe flows was introduced by Preston.[2] The method has been extended to external boundary layers[3] and to compressible isobaric[4] and nonisobaric[5] flows. The Preston-tube method is based on the assumption that there exists a valid velocity distribution law for the turbulent boundary layer (or pipe flow) in which one of the similarity parameters is the local skin friction. The mixing length derived law of the wall velocity profile satisfies this condition and for compressible, adiabatic flows may be expressed in functional form as

$$\Phi^* = (\Phi_1/\sigma^{1/2}) \sin^{-1} [\sigma^{1/2}(\Phi/\Phi_1)] = f(\eta) \qquad (1)$$

where

$$\eta = \frac{\rho_w u_\tau y}{\mu_w} \qquad \Phi = \frac{U}{U_\tau} \qquad \sigma = \frac{[(\gamma-1)/2]\, M_1^2}{1 + [(\gamma-1)/2]M_1^2}$$

With the functional relationship established, it can be seen that a single velocity measurement at some known position y above the wall will permit the solution of Eq. (1) for the friction velocity U_τ and hence the local shear stress at the wall τ_w.

The Preston-tube data in the high Reynolds number tests were obtained from the total-pressure probe reading at the wall position just prior to the survey across the boundary layer. Two different evaluations of the function $f(\eta)$ were used in the data reduction, the empirical tabulation presented by Coles[6] and the expression

$$f(\eta) = 2.5 \ln\eta + 5.5 \qquad (2)$$

from the classical mixing length development. The results are presented in Fig. 1 in the form of the ratio of the skin-friction coefficient determined by the Preston-tube method C_{f_P} to that measured by the skin-friction balance (as described in Ref. 1) C_{f_B}. It is noted that there is slightly better agree-

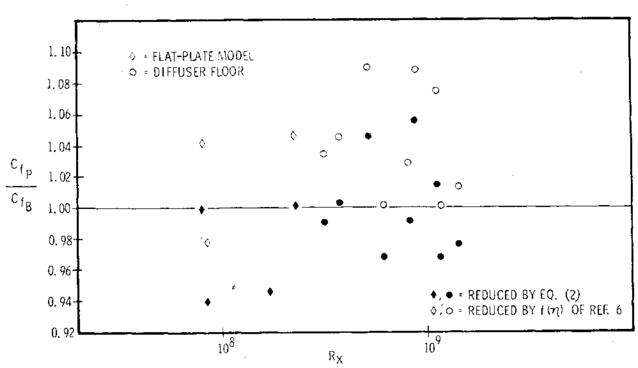

Fig. 1 Comparison of surface probe measurements with skin-friction balance measurements.

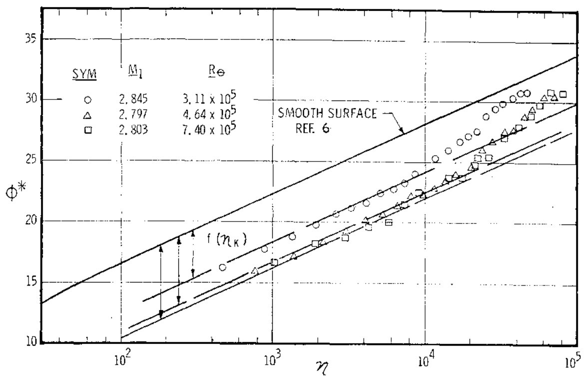

Fig. 2 Law of wall velocity profile with grain roughness, $K = 0.004$ in.

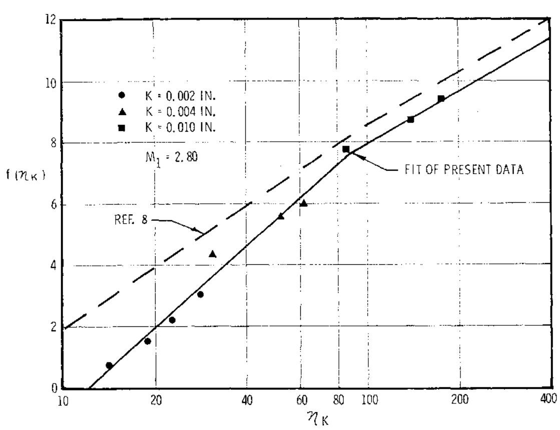

Fig. 3 Roughness function with uniform grain-type roughness.

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Table 1 **Roughness function constants, $f(\eta_k) = a_i \ln\eta_k + b_i$**

Zone	Ref. 2			Present data		
	η_k	a_i	b_i	η_k	a_i	b_i
Rough	$100 < \eta_k$	2.50	−3.00	$85 < \eta_k$	2.50	−3.85
Transition	$5.00 < \eta_k < 100$	2.84	−4.58	$12 < \eta_k < 85$	3.82	−9.45
Smooth	$\eta_k < 5$	0	0	$\eta_k < 12$	0	0

ment with the use of Eq. (2), but it must be pointed out that the Coles function normally would give better results if the measuring position were in the transition region between the laminar sublayer and fully turbulent portions of the boundary layer. It can be seen that the Preston-tube method can be considered accurate within ±6% for the test condition range of these data.

The series of tests[7] investigating the effects of grain-type surface roughness made use of the wind-tunnel diffuser floor surface plates, the skin-friction balance, and the total-pressure survey system, which were described in Ref. 1. These roughness data also were obtained at $M = 2.8$. Grain-type roughness was used in order that the results of these tests could be used to evaluate the extension of the Nikuradse roughness function[8] to the high Reynolds number compressible flow situation.

Three different grades of sand grain were tested. The mean diameters of the three grades, as determined by microscopic measurement, were 0.002, 0.004, and 0.010 in., respectively. The sand grains were bonded to the diffuser floor. The roughness covered the entire 4-ft width of the floor for a distance of 9 ft upstream of the instrumentation station. The skin-friction balance and total-pressure survey systems were installed side by side, offset equal distances from the tunnel longitudinal centerline.

As is the case with aerodynamically smooth surfaces, turbulent boundary layers over uniformly roughened surfaces exhibit certain velocity similarity characteristics. These characteristics are illustrated best by the defect law and law of the wall velocity profiles. The velocity defect law profiles of the present data indicated no effect of the surface roughness which was the expected result. Also, as expected, the law of the wall velocity profiles reflected the effect of the roughness. Figure 2 presents some profiles typical of the data obtained. It can be seen that, for the fully turbulent portion of the boundary layer over a surface with uniformly distributed roughness, the right-hand side of Eq. (1) can be expressed as $f(\eta) - f(\eta_k)$, where $f(\eta_k)$ is the so-called roughness function. In Ref. 8, Fenter correlated the roughness function as determined from the classical Nikuradse pipe flow experiments with the expression

$$f(\eta_k) = a_i \ln\eta_k + b_i \qquad \text{where} \qquad \eta_k = \rho_w U_\tau k/\mu_w \qquad (3)$$

Figure 3 presents a comparison of the values of the roughness functions of the present data as obtained from the law of the wall profiles with the correlation of Ref 8. Table 1 presents the values of the a_i and b_i coefficients of Eq. (3) from Ref. 8 and from the present data. It is noted that the present data indicate that a higher degree of roughness can be permitted and still maintain an aerodynamically smooth surface, i.e., $f(\eta_k) = 0$. The velocity profile data and skin-friction measurements show the expected trends; however, the roughness function determined from the data is not in agreement with the roughness function as previously determined in incompressible pipe flow. This difference presumably is caused by a compressibility effect; however, other compressible data exist at lower Reynolds numbers that show reasonable agreement with the incompressible roughness function. Since tests were made at only one Mach number, further investigations to obtain additional data at other Mach numbers are considered necessary in order to resolve this problem.

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Very-Large-Deflection Behavior of Corrugated Strips

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Introduction

Few techniques have been developed for analyzing the very-large-deflection behavior of thin plate and shell structures, except for some very simple cases. Barton[1] and Bisshopp and Drucker[2] have obtained results for a cantilever beam subjected to a concentrated end load, based on the Bernoulli-Euler equation. Rhode[3] obtained similar results for the case of a uniformly distributed load.

In the present note, a technique is described† for handling the very-large-deflection behavior of a corrugated strip in terms of the solution to a curved cantilever beam with an end load (Fig. 1). A stepwise incremental solution is obtained for a modified Bernoulli-Euler equation, which permits both elastic and plastic deformation of beams of arbitrary shape. The technique involves incremental loads as well as incremental elements of the structure. For each incremental load application, the corresponding deformation of the curved

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† A more complete discussion is given in Refs. 4 and 5.